



Quantifying the Innovation-Scale Trade-off in United States Environmental Sustainability: An Offset-Ratio Reading of Environmental Patenting and the Ecological Footprint

OPEOLUWA SEUN OJEKEMI

Department of Engineering Management,

University of Mediterranean Karpasia Northern Cyprus, Mersin 10, Turkey

Abstract

Empirical work on eco-innovation almost always stops at the sign of an elasticity: a negative, statistically significant coefficient on green innovation is read as evidence that technology helps the environment. That verdict is purely qualitative, and it leaves the operative policy question unanswered, since knowing that innovation lowers environmental pressure says nothing about whether innovation is expanding fast enough to cancel the pressure generated by the growth occurring alongside it. This paper proposes a correction requiring no new estimation. In a log-log cointegrating system the estimated coefficients are constant elasticities, so their ratio is itself a structural, unit-free quantity. We define the offset ratio as the output elasticity divided by the magnitude of the innovation elasticity, and show that it measures the proportional expansion in environmental patenting needed to hold the ecological footprint constant against a one percent expansion in real output. Applied to Fourier-ARDL estimates for the United States over 1970-2018, the construct yields an offset ratio of 0.49: every percentage point of output growth must be matched by roughly half a percentage point of growth in the environmental share of patenting. A delta-method interval places the ratio between approximately zero and one, so it should discipline magnitudes rather than fine calibration. We further show that the energy elasticity, conventionally reported as an insignificant null, carries an implied interval so wide that it is better described as uninformative.

Keywords: eco-innovation; ecological footprint; offset ratio; Fourier ARDL; environmental patents; United States

1 Introduction

The tension between economic expansion and environmental preservation has an orthodox resolution: grow the technology faster than the economy. If cleaner production methods diffuse quickly enough, output can rise without the accompanying rise in resource throughput, and the conflict between prosperity and ecological limits becomes a question of timing. A large applied literature has been assembled around this proposition, and much of it is supportive. Studies covering single countries and broad panels alike report that measures of technological innovation carry negative coefficients when regressed on indicators of environmental damage [Rout et al., 2022, Guan et al., 2022, Alola and Adebayo, 2022]. What that literature reports, however, is a sign. A negative and significant coefficient establishes direction, and direction alone is a weak basis for policy. Consider a government told that its green innovation effort reduces environmental pressure. It still does not know whether the effort is adequate, because adequacy is a comparison rather than a property of one coefficient. The relevant question is whether innovation is expanding fast enough to neutralise the environmental cost of the growth happening at the same time, and no amount of significance testing on a single parameter answers it. A country could satisfy every conventional test in the literature, report a strongly negative innovation elasticity, and still be losing ground.

This paper closes that gap with an observation about the algebra of the models already being estimated. When environmental damage, innovation, output and energy use enter a cointegrating



relationship in logarithms, the estimated coefficients are constant elasticities, and elasticities are unit-free, so their ratio is unit-free as well. Here that ratio has a direct reading. We define the offset ratio $\theta = \beta_{\text{GDP}}/|\beta_{\text{POET}}|$ as the proportional increase in the environmental share of patenting required to hold the ecological footprint unchanged when real output rises by one percent. The quantity converts a qualitative verdict into a sufficiency benchmark, and it is computable from any published pair of log-log elasticities without re-estimating anything. We apply the construct to the United States over 1970-2018, building on the estimates of Kirikkaleli et al. [2023], who examined the effect of patents on environmental technologies on the ecological footprint using a Fourier-augmented ARDL procedure and a Fourier Toda-Yamamoto causality test, controlling for real output and primary energy consumption. That study established the signs; the present paper does not restate its result as new. The contribution begins where the antecedent ends, by asking what those magnitudes imply for the pace of innovation the country would have to sustain. The answer is $\theta \approx 0.49$.

Five contributions follow. We define the offset ratio formally and derive it as the slope of an iso-footprint frontier in growth space. We attach uncertainty to it through the delta method, using standard errors implied by reported coefficients and p -values, and state the zero-covariance assumption this requires rather than burying it. We benchmark the required ratio against the patenting expansion the United States actually realised. We reinterpret the energy coefficient, whose implied interval is so wide that treating it as a null is a misreading. And because the construct needs only two published numbers, it offers a way of re-reading an existing literature rather than adding one more estimate to it.

2 Related Work

Three strands converge here, each stopping short of the sufficiency question for a different reason.

2.1 Footprint, output and energy

The ecological footprint became a common dependent variable once it grew clear that carbon dioxide, while convenient, captures only the atmospheric dimension of environmental pressure. The footprint aggregates the biologically productive area needed to supply a population's consumption and absorb its waste [Wackernagel and Rees, 1998], and so registers land and water degradation that emissions series miss [Dogan et al., 2020]. Evidence on the output elasticity is unusually consistent. Positive and significant estimates recur across estimators and samples: in a 22-country panel [Uddin et al., 2016], across developing economies [Jahanger et al., 2022], and in wide cross-sections [Wang et al., 2022]. The scale channel is well identified. The energy channel is not. Studies report positive effects [Kongbuamai et al., 2021], negative effects [Destek and Sinha, 2020, Salari et al., 2021], and effects that depend on which part of the conditional distribution is examined [Shahzad et al., 2021]. The heterogeneity is usually attributed to differences in energy mix, and we return to it in Section 6. What matters here is that none of these studies asks what rate of countervailing effort its own estimates imply. The environmental cost of growth is treated as a burden to be measured, not a target to be offset.

2.2 Eco-innovation and environmental quality

Work connecting innovation to environmental outcomes is far less settled. The estimated effect ranges from strongly negative through statistically null to positive, depending on country, period and proxy. Reductions are reported for BRICS economies [Rout et al., 2022], the G-10 [Guan et al., 2022] and the Nordic region [Alola and Adebayo, 2022], while no detectable effect appears for big emerging markets [Destek and Manga, 2021] and a footprint-increasing effect appears for Pakistan [Chunling et al., 2021]. Patent counts are the usual proxy. They are imperfect, since a granted patent records an invention rather than its deployment, but they are available on a consistent



basis over long horizons, which alternatives generally are not [Serener et al., 2022]. For the United States the patent-based evidence is comparatively thin, despite the country combining a large environmental technology portfolio with persistently high emissions [Tsai et al., 2016]. The methodological point stands regardless of where a given study lands. All of them terminate at sign and significance, so the question of whether the observed pace of innovation is adequate is never posed.

2.3 Smooth structural change and Fourier estimators

Long macroeconomic samples contain regime change, and conventional cointegration procedures require the analyst to specify how many breaks occurred and when. Pre-specification of this kind is fragile, and it becomes untenable when change is gradual rather than abrupt. The Fourier approach sidesteps the problem by approximating the unknown deterministic component with a few low-frequency trigonometric terms, capturing breaks of unknown form, number and date without any date search [Enders and Lee, 2012, Ludlow and Enders, 2000]. Frequency selection by minimum residual sum of squares is standard [Christopoulos and Leon-Ledesma, 2011, Omay, 2015]. The device has been carried into ARDL bounds testing [Yilanci et al., 2020], which extends the procedure of Pesaran et al. [2001] and its bootstrap refinement [McNown et al., 2018], and into Toda-Yamamoto causality testing [Alper et al., 2022].

This literature is estimator-focused, and understandably so. It establishes that Fourier augmentation delivers more reliable inference on break-prone samples and then stops, because delivering coefficients is what an estimator paper is for. It does not ask which derived functions of a cointegrating vector are worth reporting alongside the coefficients. Taken together, one literature measures the cost of growth, a second measures the effect of innovation with mixed success, and a third supplies the machinery that makes both measurable over long samples. No strand takes the final step of dividing one of its own estimates by another to ask whether the second is large enough to cancel the first.

3 Analytical Framework: The Offset Ratio

3.1 From elasticities to a trade-off rate

Let EF_t denote the ecological footprint, $POET_t$ the environmental share of patenting, GDP_t real output and EC_t primary energy consumption. The long-run relationship, augmented with a single-frequency Fourier pair to absorb smooth structural change, is

$$\ln EF_t = \alpha + \beta_{POET} \ln POET_t + \beta_{GDP} \ln GDP_t + \beta_{EC} \ln EC_t + \gamma_1 \sin\left(\frac{2\pi kt}{T}\right) + \gamma_2 \cos\left(\frac{2\pi kt}{T}\right) + \varepsilon_t, (1)$$

where k is the selected frequency, T the sample size and t the time index. Because both sides are in logarithms, each β is a constant elasticity: β_{GDP} is the percentage change in the footprint associated with a one percent change in output, and likewise for the others. Two features of (1) make a ratio of coefficients meaningful. Elasticities are dimensionless, so the ratio inherits units from neither variable and is invariant to any rescaling of EF , $POET$ or GDP . Both elasticities are also long-run objects defined on the same cointegrating vector, so they are commensurable in a way that short-run or differently-conditioned estimates would not be.

3.2 The iso-footprint frontier

Hold energy use and the deterministic terms fixed and totally differentiate (1). The condition for the footprint to remain unchanged is $d \ln EF = \beta_{POET} d \ln POET + \beta_{GDP} d \ln GDP = 0$, which rearranges to

$$\frac{d \ln \text{POET}}{d \ln \text{GDP}} = -\frac{\beta_{\text{GDP}}}{\beta_{\text{POET}}} = \frac{\beta_{\text{GDP}}}{|\beta_{\text{POET}}|} \equiv \theta, \quad (2)$$

the second equality holding whenever $\beta_{\text{POET}} < 0$ and $\beta_{\text{GDP}} > 0$, the empirically relevant configuration. We call θ the offset ratio.

Equation (2) traces a ray through the origin in the space of growth rates, shown in Figure 1. Points above the ray deliver net environmental improvement; points below deliver net deterioration despite innovation being present and effective. The frontier makes explicit what the sign convention hides: an economy can have a genuinely negative innovation elasticity and still slide into the deterioration region, simply because it is growing faster than it is innovating.

Three readings follow. A value below one means innovation is a comparatively efficient lever, since the patent share need not grow as fast as output to hold the line; a value above one implies that innovation alone is an implausible strategy. And the decision rule for any economy is a comparison: letting ρ denote the realised ratio of patent-share growth to output growth, the footprint improves if $\rho > \theta$ and worsens if $\rho < \theta$.

3.3 Uncertainty and scope conditions

Since θ is a ratio of two estimated quantities it inherits their sampling variability. A first-order Taylor expansion gives the delta-method approximation

$$\widehat{\text{Var}}(\hat{\theta}) \approx \hat{\theta}^2 \left[\frac{\widehat{\text{Var}}(\hat{\beta}_{\text{GDP}})}{\hat{\beta}_{\text{GDP}}^2} + \frac{\widehat{\text{Var}}(\hat{\beta}_{\text{POET}})}{\hat{\beta}_{\text{POET}}^2} - \frac{2\widehat{\text{Cov}}(\hat{\beta}_{\text{GDP}}, \hat{\beta}_{\text{POET}})}{\hat{\beta}_{\text{GDP}}\hat{\beta}_{\text{POET}}} \right]. \quad (3)$$

The covariance term is unavailable, because published papers in this literature report coefficients and p -values but not variance-covariance matrices. We therefore set it to zero. This is an assumption, not a result, and it is the single most consequential simplification in the paper. Its direction is knowable: the two regressors are positively correlated in levels over the sample, so their coefficient estimates are likely negatively correlated, which would make the omitted term positive and our reported interval too narrow rather than too wide. The interval should be read as a lower bound on uncertainty.

Two scope conditions govern when θ is interpretable. The system must be cointegrated, since otherwise the coefficients are not long-run objects and the total differential has no equilibrium meaning. And β_{POET} must be bounded away from zero: as the innovation elasticity approaches zero the ratio diverges, which is correct behaviour, because an economy whose innovation has no environmental effect cannot offset growth at any finite rate of innovating.

4 Data and Empirical Strategy

4.1 Variables and sample

The analysis covers the United States annually from 1970 to 2018, giving 49 observations. The ecological footprint is measured in total global hectares (Global Footprint Network), environmental patents as a percentage

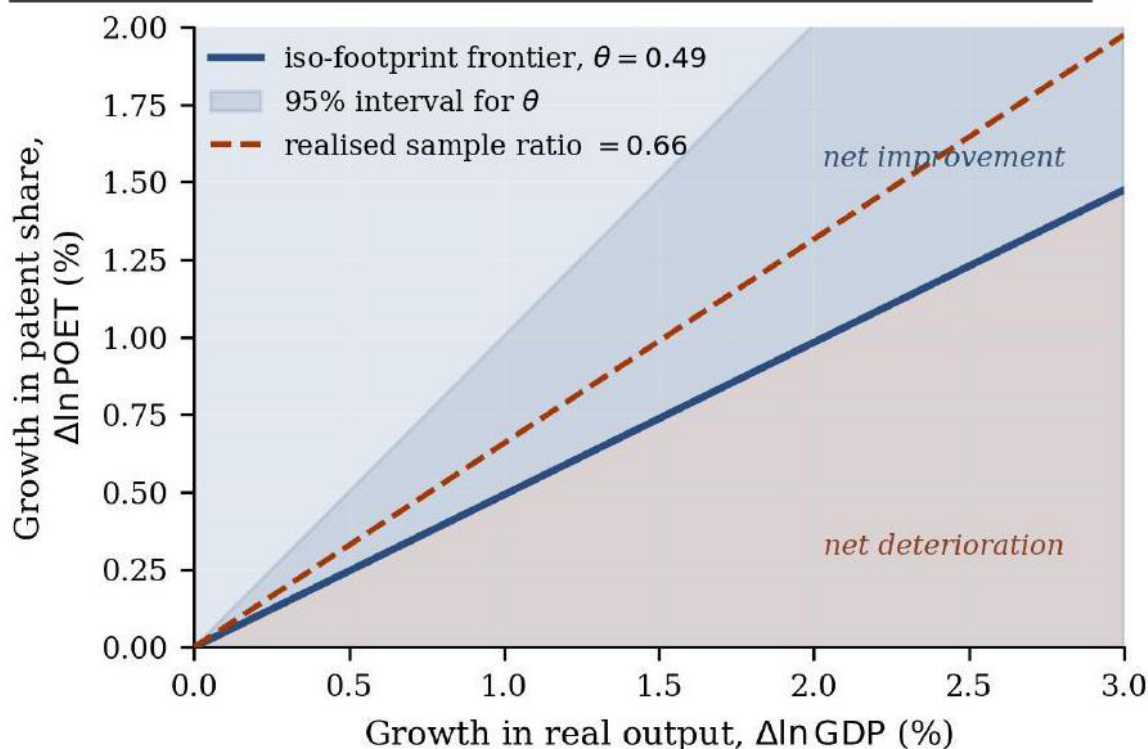


Figure 1: The iso-footprint frontier. Points on the solid line leave the ecological footprint unchanged; its slope is the offset ratio $\theta = 0.49$. The dashed line marks the ratio of realised log-ranges over the sample, which rests on an amplitude rather than an endpoint comparison.

Table 1: Sample properties, 1970-2018 ($n = 49$), all series in logarithms. Panel B reports Fourier ADF and ADF unit root tests; FADF critical values are -10.35, -7.58 and -6.35 at 1%, 5% and 10%. * denotes 1%.

Panel A: descriptive statistics

	Mean	Median	Max.	Min.	Std. dev.	Skew .	Kurt.	J-B (p)
ln EF	32.6880	32.3019	34.5361	29.9416	1.2585	-0.013	2.966	0.0037 (0.998)
ln POET	2.0850	2.0464	2.5479	1.6620	0.2426	0.543	2.225	3.7057 (0.157)
ln GDP	30.0125	30.0280	30.6230	29.2771	0.4120	-0.198	1.699	3.8538 (0.146)
ln EC	4.4221	4.4575	4.5744	4.1797	0.1237	-0.372	1.631	5.0590 (0.080)

Panel B: integration order					
Series	k	FADF	F	ADF (level)	ADF (Δ)
ln EF	10	-5.87*	12.95*	-0.63	-7.26*
ln POET	3	-2.97	3.68	-0.70	-7.23*
ln GDP	6	-2.77	4.50	4.11	-2.55*
ln EC	9	-2.74	12.34*	2.08	-5.32*

of total patents (OECD), real output in constant 2015 international dollars (World Bank), and primary energy consumption in exajoules (BP Statistical Review). All series enter in natural logarithms.

Panel A of Table 1 reports descriptive statistics. No series rejects normality at the five percent level on the Jarque-Bera test. The footprint shows the widest dispersion in log-levels and energy consumption the narrowest, which is itself informative: over five decades US primary energy use varied far less than the environmental pressure attributed to it.

A note on provenance. The primary estimates reported in Section 5 are those of Kirikkaleli et al. [2023] and are reproduced as the substrate for the derived analysis; we have not re-estimated the system. Everything labelled implied or derived, including all standard errors, the offset ratio, its interval and the sensitivity analysis, is new to this paper and was computed from the reported figures. Keeping that boundary visible matters, since the value added here lies entirely in the second category.

4.2 Fourier-augmented ARDL specification

The estimator is the Fourier-augmented ARDL bounds procedure of Yilanci et al. [2020], which embeds trigonometric terms in the conditional error-correction form of Pesaran et al. [2001]:

$$\Delta \ln EF_t = \alpha + \gamma_1 \sin\left(\frac{2\pi kt}{T}\right) + \gamma_2 \cos\left(\frac{2\pi kt}{T}\right) + \delta_1 \ln EF_{t-1} + \delta_2 \ln POET_{t-1} + \delta_3 \ln GDP_{t-1} + \delta_4 \ln EC_{t-1} + \sum_{i=1}^p \phi_i \Delta \ln EF_{t-i} + \sum_{j=0}^q \psi_j \Delta \mathbf{x}_{t-j} + u_t, \quad (4)$$

Table 2: Long-run FARDL estimates and the offset ratio. All standard errors and intervals are implied from the reported coefficients and p -values ($df = 40$), not reported by the source. In Panel B the point estimate $\hat{\theta} = 0.4916$ is invariant by construction. * 1%, ** 5%.

Panel A: long-run elasticities, dependent variable ln EF						
Regressor	Coefficient	p -value	Implied $ t $	Implied SE	Implied 95% CI	
ln POET	-4.19*	0.00	2.971	1.410	[-6.95,	-1.43]
ln GDP	2.06**	0.02	2.423	0.850	[0.39,	3.73]
ln EC	-4.47	0.40	0.851	5.254	[-14.77,	5.83]

Panel B: sensitivity of θ to the significance assumed for β_{Poet}					
Assumed p Implied SE (β_{Poet})		SE($\hat{\theta}$)	95% CI for θ		
0.005		1.410	0.262	[-0.02, 1.00]	
0.001		1.180	0.246	[0.01, 0.97]	
0.0005		1.106	0.241	[0.02, 0.96]	
0.0001		0.970	0.233	[0.04, 0.95]	

with $\mathbf{x}_t = (\ln \text{POET}_t, \ln \text{GDP}_t, \ln \text{EC}_t)'$. Lag orders are chosen by the Akaike criterion and the frequency k by minimising the residual sum of squares, following Christopoulos and Leon-Ledesma [2011] and Omay [2015]; a single frequency is used, as recommended by Ludlow and Enders [2000]. Cointegration is judged by three statistics rather than one: an overall F_A test on all level coefficients, a t test on the lagged dependent variable, and the F_B test of McNown et al. [2018] on the lagged regressors. Requiring all three to reject guards against the degenerate cases that can make a single bounds statistic misleading. Critical values are bootstrapped. Causality is then assessed with a Fourier Toda-Yamamoto procedure [Alper et al., 2022].

5 Results

5.1 Integration order and cointegration

Panel B of Table 1 reports unit root tests. The Fourier trigonometric terms are individually insignificant for $\ln \text{POET}$ and $\ln \text{EC}$, so for those series the conventional augmented Dickey-Fuller test is applied as Enders and Lee [2012] recommend. All four series are integrated of order one. Since the bounds procedure requires only that the dependent variable be stationary in first differences, this configuration permits testing. The bounds test selects $k = 1$. All three statistics clear their one percent bootstrap critical values: $F_A = 13.41$ against 10.29, $|t| = 6.41$ against 5.52, and $F_B = 8.97$ against 8.61. The margin on F_B is the narrowest, which is worth noting, but the verdict does not rest on any single test.

5.2 Long-run elasticities

Table 2 reports the long-run estimates together with the standard errors we imply from them. The patent-share elasticity is -4.19 and significant at one percent; the output elasticity is 2.06 and significant at five percent; the energy elasticity is -4.47 and not significant. Implied standard errors are obtained by inverting the reported two-tailed p -values against a t distribution with $49 - 9 = 40$ degrees of freedom, the nine parameters counting the constant, trend, two trigonometric terms, three long-run regressors and the short-run lag terms. The p -value for $\ln \text{POET}$ is reported only to two decimals as 0.00, so we adopt $p = 0.005$, the largest value consistent with that rounding. The choice is deliberately conservative: it produces the widest standard error the reported figure permits, and hence the most cautious interval downstream.

Figure 2 plots these intervals. Two features drive everything that follows. The patent elasticity is roughly twice the output elasticity in magnitude, which will place θ below unity. And the energy interval is enormous, spanning -14.77 to 5.83; Section 6 takes up what that means.

5.3 The offset ratio and its uncertainty

Substituting into (2) gives $\hat{\theta} = 2.06/4.19 = 0.4916$. Each percentage point of real output growth requires the environmental share of US patenting to expand by about 0.49 percent for the ecological footprint to stay flat. Put for a policy audience: the country's green innovation effort has to grow at roughly half the rate of its economy simply to break even. The delta-method standard error from (3), with the covariance term set to zero, is 0.262 , giving a 95% interval of [-0.02, 1.00]. Because the p -value on $\ln$ POET was rounded in the source, Panel B of Table 2 records

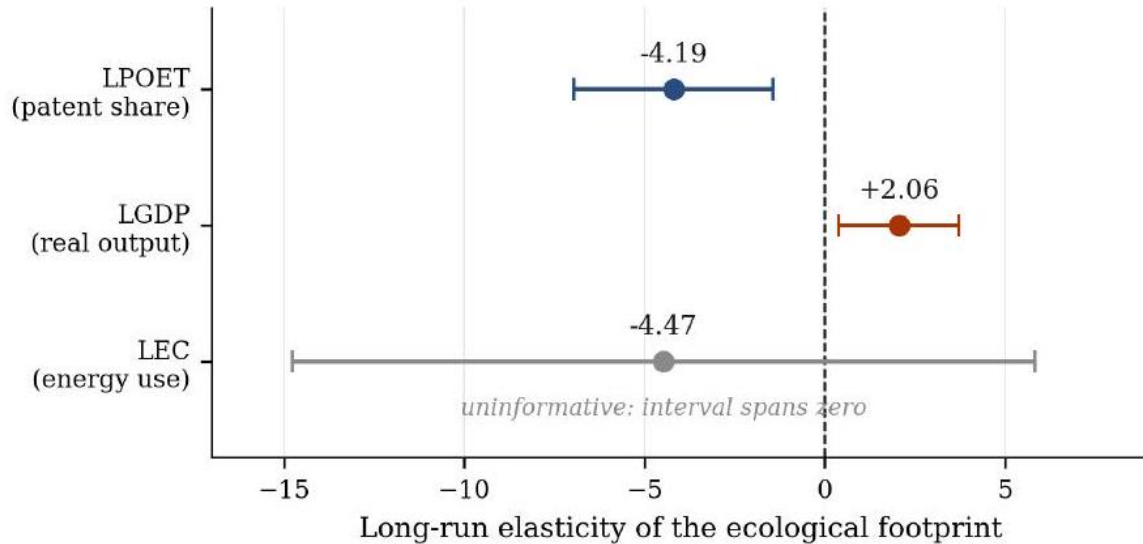


Figure 2: Long-run elasticities with 95% intervals implied by the reported coefficients and p -values ($df = 40$). The patent-share elasticity exceeds the output elasticity in magnitude, placing θ below unity; the energy interval spans zero by a wide margin.

Table 3: Fourier Toda-Yamamoto causality and residual diagnostics. ** 5%, *** 10%.

Panel A: causality			Panel B: diagnostics		
Direction	Stat.	p	Test	F	p
$\ln$ POET $\rightarrow$ $\ln$ EF	8.79***	0.07	Breusch-Godfrey LM	0.49	0.61
$\ln$ GDP $\rightarrow$ $\ln$ EF	10.46**	0.03	Breusch-Pagan-Godfrey	1.06	0.41
$\ln$ EC $\rightarrow$ $\ln$ EF	12.05**	0.02	Ramsey RESET	0.39	0.69

how the interval responds to tighter assumptions about it. The point estimate is invariant by construction, and the interval excludes zero under every assumption tighter than the conservative $p = 0.005$, but it never becomes narrow: even under the most favourable assumption considered it still runs from 0.04 to 0.95. The honest summary is that θ is estimated imprecisely. It is a magnitude rather than a calibration target, and the appropriate use is to distinguish a required offset of roughly one half from one of roughly two or one twentieth. That distinction is still worth having, since it separates innovation being a plausible primary instrument from its being an implausible one.



5.4 Causality and model adequacy

Table 3 reports the Fourier Toda-Yamamoto results and the residual diagnostics. All three regressors Grangercause the footprint once smooth breaks are accommodated. The innovation channel is the least sharply identified, significant at ten percent against five percent for output and energy, which tempers rather than overturns the reading: the direction of the relationship is supported, but less firmly than the long-run coefficient alone would suggest.

The diagnostics give no cause for concern. Serial correlation, heteroskedasticity and functional form tests all fail to reject, and the source study reports CUSUM and CUSUM-of-squares plots lying inside their five percent bands throughout. This matters more here than in an ordinary application, because θ is a ratio of two coefficients from a single equation, and a misspecified equation would corrupt both.

6 Discussion

6.1 Is the required offset attainable?

A benchmark of 0.49 is only useful against something realised. The descriptive statistics permit a rough comparison. Over 1970-2018 the log-range of the patent share was 0.886 and that of real output 1.346, a ratio of $\rho = 0.658$. Since $\rho > \theta$, the amplitude of variation in US environmental patenting was of the same order as, and somewhat larger than, the offset the estimated elasticities require. On the frontier in Figure 1 the realised ray sits above the iso-footprint line. The comparison should not be pushed hard, and we would rather state its weakness plainly than let it carry more weight than it can bear. A range is the distance between a maximum and a minimum, not a net change from one endpoint to another, and the two coincide only for a monotone series. The patent share was not monotone: it rose through the 2000s and then fell back after 2012 [Kirikkaleli et al., 2023]. A series that climbs and then partly retraces can show a large range alongside a small net change, so $\rho = 0.658$ establishes that the United States has demonstrated the capacity to expand environmental patenting at the required rate, not that it sustained that expansion. The defensible reading is that the country cleared the bar during part of the sample and then stopped clearing it.

6.2 Composition rather than volume

The energy result deserves more than the single word "insignificant" that convention assigns it. An implied interval of $[-14.77, 5.83]$ is not a finding that energy consumption does not affect the ecological footprint. It is a finding that these data, conditioned on output and patenting, cannot distinguish a large negative effect from a moderate positive one. Calling the coefficient null asserts something the estimate does not support. Why so uninformative? The descriptive statistics offer a clue: the log-range of energy consumption is 0.395, against 1.346 for output. US primary energy use was close to flat across five decades in which output more than tripled and the footprint moved substantially. A regressor that barely varies cannot explain much of a dependent variable that varies a great deal, and collinearity with output compounds the problem.

This suggests reading the whole system in composition rather than volume terms. What moved the US footprint was not how much energy the economy consumed but what kind of economy was consuming it, and the patent share proxies exactly that compositional dimension. The interpretation also helps explain the sign instability running through the energy-footprint literature [Kongbuamai et al., 2021, Destek and Sinha, 2020, Salari et al., 2021]: if the operative variable is composition, studies that measure volume will recover whatever sign the local correlation between the two happens to take, which will differ across countries and periods. We



offer this as an interpretation consistent with the evidence rather than a tested proposition, since testing it would require disaggregated energy-mix data the present setup does not contain.

6.3 Limitations

Several constraints bound what can be claimed. The standard errors underlying every interval here are implied from reported p -values, not taken from a reported variance-covariance matrix; rounding in the source, particularly the $p = 0.00$ entry, forced an assumption. We chose the conservative end and reported the sensitivity, but the intervals remain reconstructions. The delta-method interval additionally assumes zero covariance between the two elasticity estimates, and as argued in Section 3 the likely sign of the omitted term means the true interval is probably wider than the one reported. Access to the estimated covariance matrix would remove that assumption entirely, and we would encourage authors in this literature to report it. The realised-variation benchmark compares ranges rather than endpoint changes. A properly constructed test would compare ρ to θ period by period using the annual series, which the published descriptive statistics do not permit.

The results are also single-country and single-proxy. Patent shares record invention rather than deployment, and a patent never commercialised contributes to POET without abating anything. Finally, the constant-elasticity form rules out threshold effects: if the marginal environmental return to patenting diminishes as the patent share rises, θ would be a function of the level of POET rather than a constant, and the frontier in Figure 1 would bend. A nonlinear or regime-switching specification could test this directly.

7 Conclusion and Policy Implications

The eco-innovation literature reports signs. This paper argues that the signs already reported contain more information than has been extracted from them, because in a log-log cointegrating system the ratio of two elasticities is itself a structural quantity with a policy reading. The offset ratio θ measures how fast green innovation must grow to hold environmental pressure constant against economic growth, and it requires no new estimation, only division. For the United States over 1970-2018 that ratio is 0.49. Environmental patenting must expand at roughly half the rate of real output for the ecological footprint to hold steady, and faster for it to fall. Three implications follow. Because $\theta < 1$, innovation is a credible primary instrument rather than a supporting one: the required expansion is demanding but not arithmetically out of reach, and the sample shows the country achieving comparable rates for part of the period. The post-2012 decline in the patent share is a more serious signal than a reading focused on the negative coefficient alone would register, since under a constant θ a falling patent share moves the economy below the frontier while output continues to grow; sustaining the share is a distinct policy objective from raising it, and both are needed. And the wide interval argues against treating 0.49 as a target to be hit precisely. It is a magnitude that should discipline the ambition of innovation policy, not calibrate it. The construct travels. Any study reporting a log-log pair of innovation and output elasticities on a cointegrated system can compute its own θ and, where a variance-covariance matrix is available, a properly specified interval around it. Applying the diagnostic across the existing literature would show whether required offsets cluster near one half or vary with income level, energy mix and innovation system. Three extensions seem most promising: recasting the ratio for load capacity factor formulations of environmental quality [Pata and Isik, 2021], which measure ecological supply alongside demand; disaggregating patents by technology field; and estimating θ on a cross-country panel to establish whether it is a national parameter or a common one.



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